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A MATHEMATICAL MODEL FOR ASSESSING  
WEAPONS EFFECTS FROM GELATIN PENETRA-  
TION BY SPHERES

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### SUMMARY

A mathematical model of the penetration of 20% gel by nondeforming spheres is derived and checked against experimental data. The model, which is found to be a generalized form of Resal's Law, is:

$$F = C_v A \frac{\mu}{b} v + C_I \rho A v^2$$

where

$F$  = force retarding the sphere

$A$  = presented area of the sphere

$v$  = velocity of the sphere

$b$  = thickness of the liquid boundary layer  
surrounding the moving sphere

$\mu$  = Coefficient of viscosity of the boundary layer

$\rho$  = density of the gelatin

$C_v, C_I$  = dimensionless constants of proportionality

This equation has the solution:

$$v = \left( v_0 + \frac{C_v \mu}{C_I \rho b} \right) \exp \left( -C_I \rho \frac{A}{m} x \right) - \frac{C_v \mu}{C_I \rho b}$$

where

$v_0$  = striking velocity

$m$  = mass of the sphere

$x$  = penetration depth

If the centimeter-gram-second (cgs) measure system is used,

$$C_I = 0.15 \text{ (dimensionless)}$$

$$C_v \mu / b = 4705 \text{ gm/cm}^2 \text{ sec}$$

$$\rho = 1.07 \text{ gm/cm}^3$$

Other parameters are dependent on the projectile (area, mass, striking velocity). From these equations one may derive other relationships (e.g., energy deposit versus depth of penetration) which are useful in accessing weapons effects on personnel targets.

## PREFACE

The work described in this report was authorized under Task Nos. 1W062116A08103, 1W062116A09200, 1W52607AD1201, 1W562607AD1403, 1W562607AD1701, 1W562603A00406, 1W562603A00302, 1X562603A31200, 1W564602D02802, and 1W662708A01101. This analysis was done in calendar year 1972. The data were generated over a period of several years.

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## A MATHEMATICAL MODEL FOR ASSESSING WEAPONS EFFECTS FROM GELATIN PENETRATION BY SPHERES

### I. INTRODUCTION.

For many years the majority of ballistics testing of antipersonnel projectiles has been in 20% gelatin gel<sup>1,2</sup>. Gelatin is used for two reasons; (1) it simulates the retarding properties of skeletal muscle fairly well, and (2) it may be cast into transparent blocks. The latter property is important because it allows one to take high-speed motion pictures of the penetrating missile for retardation studies. Also, one may later observe the permanent damage without disrupting the block.

However, even the highly efficient testing now possible with gelatin has two major drawbacks. First, laboratory experiments are expensive when compared to analytical paper studies - a firing program occupies large amounts of time and manpower, and the resources spent apply only to the weapon tested. Second, one must have designed and fabricated the projectile in order to test it.

Such a "cut-and-try" method involving redesigning and retesting is a very inefficient way to build weapons. The ideal evaluative procedure is one in which the gelatin penetration/retardation performance of a projectile could be predicted from its physical characteristics. Conversely, the same models which allow prediction of performance could be used to design projectiles possessing the desired terminal, soft-target effects within the constraints placed on the weapon system.

An integral part of most weapons effectiveness analyses - and therefore the most common measure of antipersonnel effect - is the probability of incapacitating an infantry soldier, given a random hit, or  $P(I/H)$ . It has recently been shown that  $P(I/H)$  is closely correlated to the expected kinetic energy deposit (EKE) in the "average" soldier struck at random by a particular projectile.<sup>3</sup> The EKE from random hits on actual enemy soldiers cannot be measured; but EKE may be approximated by:

$$EKE = \int_0^{x_{\max}} F(x)P(x)dx \quad (1)$$

where  $F(x)$  is the retarding force on the projectile as a function depth of penetration,  $x$ , into a 20% gelatin block,  $P(x)$  is the probability that the projectile would still be within the "average" soldier at depth  $x$ , given a random hit, and  $x_{\max}$  is the maximum gelatin penetration depth of the projectile.  $F(x)$  is usually calculated from time-penetration data derived from high-speed movies of gelatin impacts. However, if the gelatin retardation could be predicted from a mathematical model, then the predicted  $F(x)$  could be used to calculate EKE. Since  $P(x)$  is already known, EKE's derived in this manner would require no firing. In fact, EKE's could be predicted for purely hypothetical projectiles.

The purpose of this report is to develop a generalized mathematical model of gelatin retardation for the simplest type of projectile, a nondeforming sphere. The use of a nondeforming sphere for the pilot model eliminates the orientation dependence and keeps missile characteristics constant during penetration.

## II. RESULTS.

In this section we will develop a mathematical model of the retardation of a nondeforming sphere in 20% gelatin. The gelatin is thixotropic; that is, it may be transformed from its elastic solid state (gel) to a viscous liquid state (sol) by the application of pressure. Thus the front surface of the penetrating sphere is presumed to be in contact with a thin layer of viscous liquid back to the point where the medium separates from the sphere and a cavity begins to form. The last bit of penetration, where the velocity (and pressure) drops below that which will liquefy the gel, is shortened because the sphere is penetrating an elastic solid instead of a viscous liquid. This effect will be neglected because the amount of energy remaining in the sphere at that point is negligible compared to the striking energy and, for the most part, it is the deposit of energy with which we are really concerned.

The relevant parameters associated with both the sphere and the gelatin, and the units of mass M, length L, and time T in which they are expressed, are as follows:

For gelatin:

- Density,  $\rho$  (M/L<sup>3</sup>)
- Coefficient of viscosity,  $\mu$  (M/LT)
- thickness of the liquid boundary layer, b (L)
- retarding force, F (ML/T<sup>2</sup>)

For the sphere:

- mass, m (M)
- presented area, A (L<sup>2</sup>)
- velocity, v (L/T)
- retarding force, F (ML/T<sup>2</sup>)

Notice that the common parameter is the retarding force. It determines, of course, the deceleration of the sphere and also the coefficient of viscosity and thickness of the boundary layer in gelatin.

We will use dimensional analysis to initiate the derivation of the model. First we form the dimensionless product:

$$M^0 L^0 T^0 = m^{a_1} A^{a_2} v^{a_3} \rho^{a_4} \mu^{a_5} b^{a_6} F^{a_7} \quad (2)$$

where the a's are the unknown powers to which the variables are to be raised to make the product dimensionless. If the dimensions of the seven variables are put in place of the variables in equation 1 and the powers of M, L, and T collected, the following results:

$$M^0 L^0 T^0 = M^{a_1 + a_4 + a_5 + a_7} L^{2a_2 + a_3 - 3a_4 - a_5 + a_6 + a_7} T^{-a_3 - a_5 - 2a_7}$$



Since the powers of each variable on both sides of the equation must be equal, we have three equations in seven unknowns. Three of the variables, therefore, may be expressed in terms of the remaining four. Since we are to generalize for all spheres, we choose to solve for  $a_1$ ,  $a_2$ , and  $a_3$  which are the exponents of the properties describing the sphere,  $m$ ,  $A$ , and  $v$ .

Thus 
$$a_1 = -a_4 - a_5 - a_7$$

$$a_2 = \frac{3}{2} a_4 + a_5 - \frac{1}{2} a_6 + \frac{1}{2} a_7$$

$$a_3 = -a_5 - 2a_7$$

Substitution into equation 1 yields

$$\begin{aligned} M^0 L^0 T^0 &= m^{-a_4 - a_5 - a_7} A^{\frac{3}{2} a_4 + a_5 - \frac{1}{2} a_6 + \frac{1}{2} a_7} v^{-a_5 - 2a_7} \rho^{a_4} \mu^{a_5} b^{a_6} F^{a_7} \\ &= \left( \frac{\rho A^{\frac{3}{2}}}{m} \right)^{a_4} \left( \frac{A \mu}{m v} \right)^{a_5} \left( \frac{b}{A^{\frac{1}{2}}} \right)^{a_6} \left( \frac{\frac{1}{A^2} F}{m v^2} \right)^{a_7} \end{aligned}$$

These four terms are the dimensionless variables that we were seeking.

Refer to these terms as density, viscosity, boundary layer and force terms, respectively (because each of these parameters is associated with only one term). Dimensional analysis does not give us the form of the relationship among these terms. It might be linear, exponential, logarithmic, etc. However, it is reasonable to assume that the force on the missile is primarily due to an inertial component, corresponding to the mass of the gelatin which is moved aside as the missile penetrates, and a viscous component due to the internal friction of the sol (liquid). It is also reasonable to assume that the two components are linearly additive. However, in the above dimensionless terms we have the force term plus three terms - not two. To see how to obtain an inertial and a viscous component of force, we examine the definition of viscosity.

Imagine that we have two plates of surface area  $a$ . Between these plates is a viscous liquid of coefficient of viscosity  $\mu$  (as in Figure 1). The distance separating the two plates is  $b$ . If the bottom plate is motionless and the top plate is moving with velocity  $v$ , the viscous force resisting that motion is

$$F = \mu a v / b \quad (3)$$

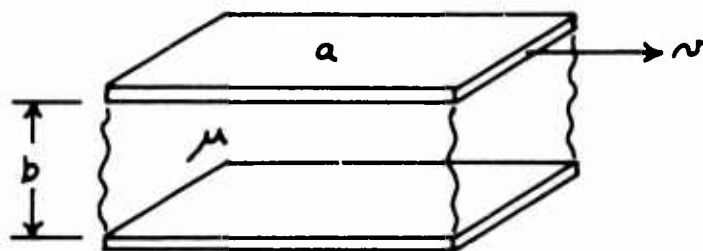


Figure 1. Shearing of a Viscous Liquid

By analogy with equation 3 we expect the dimensionless viscous force term to be directly dependent on the viscosity  $\mu$  and inversely dependent on the boundary layer thickness  $b$ . This requirement is met if we let the viscous force term be expressed by the viscous term divided by the boundary layer term. Thus the total force may now be expressed as the sum of the viscous and inertial force terms, or

$$\left( \frac{\frac{1}{A^2} F}{mv^2} \right) = C_v \left( \frac{\frac{3}{A^2} \mu}{mb} \right) + C_I \left( \frac{\rho A^{\frac{3}{2}}}{m} \right) \quad (4)$$

where  $C_v$  and  $C_I$  are proportionality constants for the viscous and inertial terms respectively. Notice that since all bracketed terms in equation 4 are dimensionless,  $C_v$  and  $C_I$  are dimensionless constants. Solving equation 4 for force gives us:

$$F = C_v \frac{A\mu}{b} v + C_I \rho A v^2 \quad (5)$$

Equation 5 is the generalized form of Resal's Law. By Newton's formulation the force is defined as the mass times the acceleration (or deceleration in this case, since it is negative); or, if  $x$  equals displacement and  $t$  equals time,

$$F = -mdv/dt = -mdv/dx \cdot dx/dt = -mv dv/dx \quad (6)$$

Combining equations 5 and 6, we get

$$dv/dx = -C_v \frac{A\mu}{mb} - C_I \rho \frac{A}{m} v \quad (7)$$

or

$$dv/dx = -K_1 - K_2 v$$

where  $K_1$  and  $K_2$  are constants. It is clear that, since the sphere is considered nondeforming,  $A$  and  $m$  are constant. The gelatin is considered incompressible, making  $\rho$  a constant and limiting the model's applicability to velocities less than the velocity of sound in the gelatin. However, as was stated before,  $\mu$  and  $b$  are not constant. Let us assume that each is strictly proportional to velocity; i.e.,

$$\left. \begin{array}{l} \mu = C_1 v \\ \text{and } b = C_2 v \end{array} \right\} \frac{\mu}{b} = \frac{C_1}{C_2}$$

The ratio  $\mu/b$  would be constant, then, and must be kept as a ratio if it is later incorporated as a constant into other expressions. Separating the variables in equation 7 and integrating from  $v = v_0$  at  $x = 0$  to  $v = V_x$  at displacement  $x$  yields

$$\int_{v_0}^{v_x} \frac{dv}{K_1 + K_2 v} = - \int_0^x dx$$

$$\frac{1}{K_2} \ln (K_1 + K_2 v) \Big|_{v_0}^{v_x} = -x$$

$$K_1 + K_2 v_x = (K_1 + K_2 v_0) e^{-K_2 x}$$

$$v_x = \left( v_0 + \frac{K_1}{K_2} \right) \exp \left( -K_2 x \right) - \frac{K_1}{K_2}$$

or

$$v_x = \left( v_0 + \frac{C_v \mu}{C_I \rho b} \right) \exp \left( -C_I \rho \frac{A}{m} x \right) - \frac{C_v \mu}{C_I \rho b} \quad (8)$$

At the maximum penetration distance,  $x_{\max}$ , where  $v_x = 0$ , equation 8 reduces to

$$x_{\max} = \frac{m}{C_I \rho A} \ln \left( 1 + \frac{C_I \rho b v_0}{C_v \mu} \right) \quad (9)$$

The dimensionless quantity  $\frac{\rho b v_0}{\mu}$  is a form of Reynold's number.

The general equation - i.e., penetration depth  $x$  as a function of time, of the form of equation 9 - is obtained by integrating equation 4 with respect to time. That is, combining equations 5 and 6 again, we get

$$-dv/dt = K_1 v + K_2 v^2$$

The variables are separated and the equation is integrated;

$$\int_{v_0}^{v_t} \frac{dv}{K_1 v + K_2 v^2} = - \int_0^t dt$$

where  $v_0$  is the initial velocity and  $v_t$  the velocity at time  $t$ . This gives us

$$\ln \left[ \frac{(K_1 + K_2 v_0) v_t}{(K_1 + K_2 v_t) v_0} \right] = -K_1 t$$

which we solve for  $v_t$ .

$$v_t = \frac{dx}{dt} = \frac{v_o}{\left(1 + \frac{K_2 v_o}{K_1}\right) e^{K_1 t} - \frac{K_2 v_o}{K_1}}$$

Integrate again for  $x$  as a function of time.

$$\int_0^x x dx = \int_0^t \frac{v_o dt}{\left(1 + \frac{K_2}{K_1}\right) v_o e^{K_1 t} - \frac{K_2}{K_1} v_o}$$

This yields

$$K_2 x + K_1 t = \ln \left[ \left(1 + \frac{K_2}{K_1} v_o\right) e^{K_1 t} - \frac{K_2}{K_1} v_o \right]$$

Exponentiate, then solve for  $x$ .

$$x = \frac{1}{K_2} \ln \left[ 1 + \frac{K_2}{K_1} v_o \left(1 - e^{-K_1 t}\right) \right]$$

or

$$x = \frac{\bar{m}}{C_I \rho A} \ln \left[ 1 + \frac{C_I}{C_v} \frac{\rho b v_o}{\mu} \left(1 - e^{-\frac{C_v \mu A}{b m} t}\right) \right] \quad (10)$$

Notice that equation 10 reduces to equation 9 at  $t = \infty \Rightarrow x = x_{\max}$ . From equation 9 it may be seen that if the above assumptions are justified a plot of  $Ax_{\max}/\bar{m}$  versus  $\ln(v_o)$  would be a smooth curve for data on any nondeforming spheres. Penetration data ( $x_{\max}$ ) were available for a variety of spheres made of steel, tungsten, and hardened lead ranging from 0.85 grains to 19 grains in mass. From the  $\ln(v_o)$  curve an estimate was made of  $C_I$ . Equation 9 was put into the following form:

$$Y = \exp \left( \frac{C_I \rho A x_{\max}}{\bar{m}} \right)^{-1} = \frac{C_I \rho b v_o}{C_v \mu} \quad (9a)$$

$C_I$  may now be considered known so  $Y$  may be calculated and plotted against  $v_o$  as in figure 2. The slope of the regression line yields an estimate of the remaining constant,  $C_v \mu/b$ , by means of the least-squares-fitted line. Estimates of the constant parameters obtained from these data are:

$$C_I = 0.15$$

$$C_v \mu/b = 4705 \text{ grams/cm}^2 \cdot \text{sec}$$

also

$$\rho = 1.07 \text{ grams/cm}^3$$

The remaining quantities, mass, velocity, etc., are variables which may be set at any value. If they are expressed in the cgs measurement system, then the results will be in that system also.

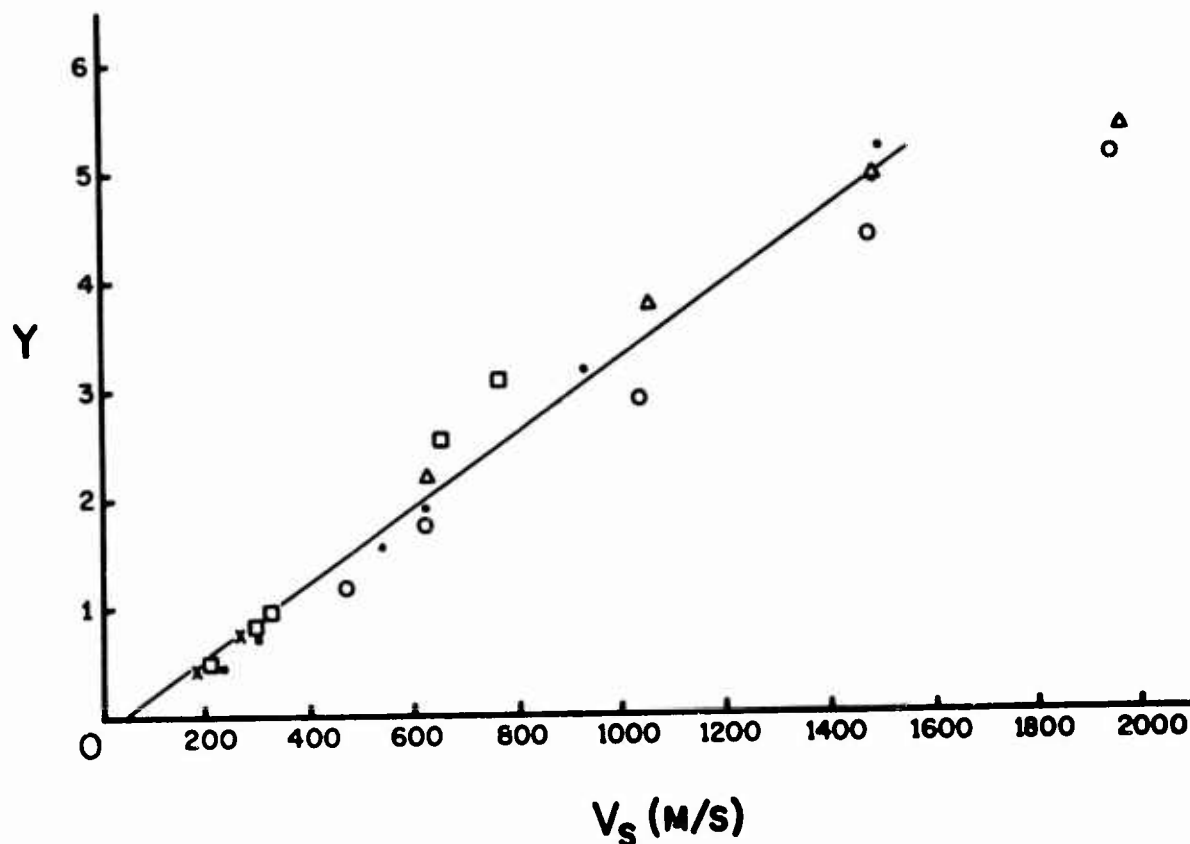


Figure 2. Scaled Penetration versus Striking Velocity

As mentioned in the introduction, any function of penetration depth or time associated with gelatin penetration by spheres may be calculated from Resal's Law. This includes energy deposit, such as EKE, or rate of energy deposit, such as might be required for more complex functions of energy distribution.

### III. CONCLUSIONS.

The generalized Resal's Law for penetration of gelatin by spheres has been shown to be sufficiently accurate to be used to derive the EKE, and therefore  $P(I/H)$ , for a sphere of any size, mass, and striking velocity. It could be used in optimizing the design of weapons such as the US

Claymore Mine (which, when detonated, launches a large number of spheres from the face of a curved slab of high explosive). These encouraging results stimulate us to explore the following possible extensions of the effort:

A. Implement a computer algorithm for deriving Resal's Law parameters from time-penetration data. This method would have two advantages over the total penetration method used in the preceding section of this report: greater accuracy because of the ability to neglect the final elastic portion of penetration, and greater generality through the ability to use data on projectiles which do not stop in the block.

B. Apply the same or a very similar model to data on cubes and irregular (but "chunky") fragments in order to calculate EKE and  $P(I/H)$  for those projectiles.

C. Extend the model to incorporate changing presented areas (and perhaps changing "constant" parameters) due to tumbling of nondeforming bullets.

The application to weapons design in accomplishing the latter two goals is obvious.

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